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The Initial Buy Quantity

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A quantitative cost model is developed to determine the initial buy quantity for a new part in a service part distribution center. New parts are introduced to the inventory throughout the year when engineering changes and when new models on the finished good items occur. Five to ten percent of the service parts are new each year. Even though there is no past history of demands on these new parts, decisions are needed on how much stock to carry initially to meet the oncoming demands for repair and maintenance. This is a critical decision on the service parts management. An example is presented to illustrate the model.

Keywords: Service parts, Buy quantity, New parts

Introduction

The initial buy quantity (IBQ) problem in service parts occurs when a quantity to buy and stock is needed for a new part with no prior demand history. The part may be a stand-alone item or it may be associated with a new finished good item or associated with an engineering change on the finished good item. Regardless, the part has not been in use before. This situation is common for service parts where the parts are stored in a distribution center awaiting the demands from customers for maintenance and repair of the finished good units. The stocking location is confronted with determining how much to buy and store on each new part of this type.

This is an old problem with no quantitative models to assist the stocking facilities of these parts. The problem on how much to stock is indeed very elusive and difficult since there is no data to base the stocking decision. Often the decision on the amount to stock is assigned to experts who review each new part and determine the quantity to buy. The experts have little if any data to back their decisions and rely mainly on their experience.

The author found only a few references on this topic in the literature. (Brown 1977) reports after the new product is introduced, three to six months pass before any demand data is available for making revisions to the initial demand estimate. (Brown 1977) also proposes two forecasts to determine how much to order initially, a minimum and a maximum. The minimum is equal to the maximum reasonable demand forecast through the lead time. The maximum is the minimum forecast through the end of the planning horizon. (Riggs and Tersine 1978) develop a technique for new product inventories using the beta distribution and subjective estimates of demands and lead times. The estimates are derived from opinions of experts such as product managers, engineers, salespeople, suppliers, production supervisors and so forth. (Hollier 1980) ranks spare parts according to a ratio of the total expected usage to a cost figure. The spare parts usage or failure rates were predicted at the design stage. (Fortuin 1984) uses the reliability of the part and the number of products containing the part that are marketed. A constant failure rate is assumed. (Lentz 1996; Lentz and Thomopoulos 1996) propose a method to build a data base to predict an introductory part's initial order quantity. This is a cost model that is based on a relationship between the previous year's introductory part demand and the cost of the current part.

Note, the methods cited above use little to no quantitative computations and -- for the most part -- the outcome strictly relies on estimates provided from experts on the parts. One of the methods does use the cost of the current part to seek a link with any parts of the past that have a similar cost; but offers no quantitative way to obtain an optimal quantity. The IBQ model presented here does use the data base to gather information from the parts of the past that are similar to the new part that is under consideration. The model is quite different from the methods noted above. This method is fully quantitative. It begins by generating a distribution of the demands (forecasts) that range from low to high possibilities. The forecasts are used to find an acceptable collection of candidate order quantities. Then, a cost model explores every combination of demands with each candidate order quantity to determine the order size that is optimal.

This paper formulates a quantitative model to find the optimal initial buy quantity. A cost model is developed and for each new part, the quantity to initially buy is selected to minimize the cost from this model. The model estimates the annual (initial year) cost of each new part based on four components: the annual order cost, the annual holding cost, the annual goodwill cost and the annual excess cost. The quantity associated with the minimum annual cost is selected as the IBQ for the part. The method used here is similar to that applied to find the optimal quantity for 'dated items' as described fully in (Thomopoulos 1990).

The notion of new-parts-of-the-past and new-parts-of-the-future is introduced. The new-parts-of-the-future are the parts where a decision on the quantity to stock is needed now. This quantity is called the initial-buy-quantity (IBQ). The new-parts-of-the-past are parts where the introduction phase on the part has already occurred and where demand data is still available (on the database) for the initial year of this introduction.

To carryout the cost model, some data searching from the new-parts-of-the-past is needed. The new-parts-of-the-past are gathered and data from their first year of demands are obtained. Also the cost-per-unit of each new-part-of-the-past is saved when available.

A key need is to recognize a set of attributes associated with each new-part-of-the-past. When possible, the attribute should identify the type of part, where and how it is used on the finished goods. The collection of attributes depends on the type of data available on the database for the parts. The attribute is needed to form a link for the new-parts-of-the-past with the new-parts-of-the-future. This way when a new-part-of-the-future is in need of an initial buy quantity, the data from the corresponding new-parts-of-the-past are used to help determine the quantity to stock.

Data from new-parts-of-the-past

To begin, a search of the database tables is needed to identify any parts where the first year of demand history is still available on the database. To accomplish, the monthly demands for the history months are needed along with the introduction date (month and year). This initial month must fall within the time frame of the history month data. Further, twelve months of data are needed including the initial month and thereby the introduction date must be 12 months prior to the end of the last month of the data available. For these parts, the first 12 months of demand history are known and denoted as ($D_1,...,D_{12}$). Also for these parts, the cost per unit (*cu*) is useful data when it is available. Another important set of information on the parts concerns the attributes. The attributes are used to identify the characteristic of the part (e.g., type or use of the part). The attributes are essential since they will act as a link between the new-parts-of-the-past with the new-parts-of-the-future. The history data and the analysis to come will relate to the attributes on the parts of the past. For each new part, the attribute is identified and the tie between the parts with the same attributes of the past are used to determine the IBQ for this new part. For convenience in this paper, the notation A is used to denote the set of attributes associated with the parts.

The average 1-month demand for a new-part-of-the-past For each new-part-of-the-past, the average 1-month demand from the initial 12 months of demands is obtained and this is denoted as d. Hence, $d = \Sigma D_t /12$ (t = 1 to 12).

Mean and standard deviation of average 1-month demands for the attribute From all the parts of the past, n will represent the number of parts with the same attribute A. These n parts represent a sample of the parts of the past with the attribute A. The average 1-month demands are gathered and labeled as $d_1,...,d_n$. This set of demands now allow computing the mean and standard deviation for the 1-month average demand for the attribute and these are here denoted as μ_d and σ_d. From the authors experience, the spread of the d entries is often skewed far to the right since there are many low values and one or a few high values. This suggests the shape is more like the lognormal distribution than as the normal distribution. For this reason, the lognormal distribution is assumed in the cost model

Average cost per unit for the attribute From the n sample parts with attribute A, not all of the parts may have data on the cost per unit. It may be that a contract with the supplier is not available and the cost on the part is not yet known. Thereby not all n parts of the attribute sample may have a known cost per unit. Let n' designate the number of parts with a cost per unit known and so the known data becomes $cu_1, \ldots, cu_{n'}$. From the parts with cost data, the average cost per unit is computed and here denoted as c.

Attribute data So for now, the data known on each attribute A consists of the mean and standard deviation of the average 1-month demands (μ_d, σ_d) and also the average cost per unit (c).

Probability points For subsequent computations, ten equally spaced probability points are selected. These are denoted as $\alpha(i)$ and are calculated in the following way: $\alpha(i) = (0.1\times(i-1) + 0.05)$ for i = 1 to 10. The calculations yield the ten equally spaced probabilities: (0.05, 0.15, ... , 0.95).

Ten average 1-month demand points Recall, the lognormal distribution is assumed for the average 1-month demands (d) of each attribute A. Of interest now is to select ten representative values of d and they are identified as $d(i)$ (i=1 to 10). For convenience, the values are called demand points and they are determined in a way where $P(d < d(i)) = \alpha(i)$ for i = 1 to 10.

(Thomopoulos and Johnson 2004) show how to transform the lognormal distribution parameters to the normal distribution parameters and these methods are used below. To find the ten demand points of $d(i)$, apply the following four steps:

1. Find μ_y, σ_y (from μ_d and σ_d) where y is the counterpart normal distributed variable that is associated with the lognormal distributed variable of d. These are computed from the relations below that use the natural logarithm, (ln):

$$\mu_y = \ln\left(\mu_d^2 / \sqrt{\sigma_d^2 + \mu_d^2}\right)$$

$$\sigma_y^2 = \ln\left((\sigma_d^2 + \mu_d^2) / \mu_d^2\right)$$

2. Find $z(i)$ from the standard normal distribution where $P(z < z(i)) = \alpha(i)$.
3. Find $y(i) = \mu_y + z(i)\sigma_y$.
4. Now $d(i) = \exp(y(i))$.

The results yield $d(i)$ where $P(d < d(i)) = \alpha(i)$. Note, for attribute A, the ten demand points here represent ten equally likey values of the average 1-month demands. These will be used subsequently.

Data for a new-part-of-the-future

Suppose a new part is now under review. The part has no demand history. But also assume, the data known on the part is the attribute A and also the lead time L. The lead time represents the time to procure the part from the supplier. This is the limited data that is available to determine the initial buy quantity.

When the cost per unit (cu) is known for the new part, $c = cu$ is used as the cost per unit in the subsequent cost model. When the cost per unit is not known for the new part, the average cost per unit (c) -- from the collection of new-parts-of-the-past with the same attribute A -- is used in the cost model.

The attribute for the new part allows the mean and standard deviation μ_d and σ_d to be retrieved. Of need here is to now find the corresponding statistical measures associated with the average lead time demand of the attribute, denoted as d_L. Because the average 1-month demand is assumed to follow a lognormal distribution, the average L-month demand also follows a lognormal distribution. For d_L, the mean becomes $\mu_{dL} = L\times\mu_d$ and the standard deviation is $\sigma_{dL} = \sqrt{L}\,\sigma_d$.

Ten average L-month demand points Ten representative values of the lead time demand are now obtained and are labeled as $d_L(i)$ where $P(d_L < d_L(i)) = \alpha(i)$ for i = 1 to 10. The values of $d_L(i)$ are found in the same way as described above for $d(i)$. The counterpart normal variable y is now associated with the lognormal variable d_L. Note, the ten demand points here represent ten equally likey values of the average L-month demands.

Ten candidate IBQ points Ten candidate values of the IBQ are now obtained. These are denoted as $q(i)$ where $q(i) = d_L(i)$ for i = 1 to 10. In this way, the IBQ for the part will be selected from one of the ten IBQ points and these points are spread along the distribution of the average lead time demand. Recall the lead time demand is assumed to follow a lognormal distribution.

Summary on data for a new-part-of-the-future Consider the data that is gathered and calculated for a new part. The new part has an attribute *A* and also a lead time *L*. The attribute is linked to the new-parts-of-the-past that also have the same attribute *A*. The data now available for the new part are the following:

A = attribute
L = lead time
c = cost per unit
$d(i)$ = ten demand points i = 1 to 10
$q(i)$ = ten candidate IBQ points i = 1 to 10.

Annual cost model

To determine an economic quantity for the IBQ, a cost model is constructed. The ten possible IBQ values determined above are assumed, and the final choice will come from one of these ten values. The purpose is to estimate the annual cost of the new part with each candidate IBQ value. The IBQ associated with the minimum cost is selected as the economic IBQ to use.

Let $K(q,d)$ = annual cost when the IBQ is q and the average 1-month demand is d. In the model, $K(q,d)$ is composed of four components as follows: $K(q,d)$ = K(order) + K(hold) + K(goodwill) + K(excess). The annual costs for each of the components are computed as below.

The cost model uses the following five parameters:

h = annual holding rate.
Co = cost per unit.
gw = goodwill rate, whereby the lost customers is estimated as $gw\times$(out-of-*stock* demands).
mu = markup rate, whereby the selling price per unit is estimated as $pu = mu\times c$.
sr = scrap rate , whereby any IBQ stock held over one year is scrapped by this rate.

Some initial computations are carried out prior to finding the cost for each of the components. These are below.

$d_{12} = 12\times d$ = average 12 month demand.
$qe = \sqrt{2d_{12}Co/ch}$ = the economic order quantity.
$t1 = q/d$ = the number of months till q is consumed. If $t1 > 12$, then set $t1 = 12$.
$t2 = qe/d$ = the number of months till qe is consumed.
$d_L = L\times d$ = average lead time demand.
$pu = mu\times c$ = the selling price per unit.

During the first 12 months, the cost model assumes q is used initially and then qe is used subsequently as needed for the remainder of the year. The four cost components are computed as below for each combination of q and d.

$$\text{K(order)} = \begin{cases} Co(1+(12-t1)/t2) & \text{if } q \le d_{12} \\ Co & \text{if } q > d_{12} \end{cases}$$

= the annual cost of ordering.

$$\text{K(hold)} = \begin{cases} c\times h/12(q/2\times t1 + qe/2\times(12-t1)) & \text{if } q \le d_{12} \\ c\times h\,(d_{12}/2 + (q- d_{12})) & \text{if } q > d_{12} \end{cases}$$

= the annual holding cost.

$$\text{K(goodwill)} = \begin{cases} 0 & \text{if } d_L \leq q \\ (d_L - q)\times pu \times gw & \text{if } d_L > q \end{cases}$$

= the cost when the q is short of the lead time demand.

$$\text{K(excess)} = \begin{cases} 0 & \text{if } q \leq d_{12} \\ (q - d_{12})c\times sr & \text{if } q > d_{12} \end{cases}$$

= the cost of holding stock in excess of one year demand.

So now the annual cost when the IBQ is q and the average 1-month demand is d is computed using the parameters, computations and equations listed above. This cost is denoted as $K(q,d)$.

It is possible now to estimate the expected cost when the IBQ = q regardless of the average 1-month demand. Note this is

$$K(q) = \Sigma_i\, K(q,d(i))\times 0.1$$

= the expected annual cost when the IBQ = q.

Recall above where ten candidate points of q have been selected. For convenience here, these are labeled as q(j) where j = 1 to 10. With the ten candidate IBQ points, it is now possible to determine the IBQ that is associated with the minimum annual cost. This is q' where $K(q') = \min\{\, K(q(j))\;\; j = 1 \text{ to } 10\,\}$. And so, the economic IBQ to use for the new part is IBQ = q'.

Example

An example is presented to illustrate how the economic IBQ is found. The example considers a new-part-of-the-future with an attribute A, lead time L = 2 months and cost per unit cu = $1356. Suppose also, six new-parts-of the-past are found with the same attribute A. The average (first year) monthly demands for these six parts are: $(d_1,\ldots,d_6)$ = (8.58, 1.00, 1.25, 1.33, 1.50, 1.08). The average and standard deviation from the six demands are computed and are denoted as $\mu_d = 2.45$ and $\sigma_d = 3.01$. Since the demands (d) are assumed to follow a lognormal distribution, the relations given earlier to find the statistics for the counterpart normal distribution (y) are now used; and yield $\mu_y = 0.442$ and $\sigma_y = 0.956$.

So now the ten demand points of d are found as described here. To accomplish, we use the ten probabilities α(i), i = 1 to 10 and the corresponding ten standard normal measures z(i), i = 1 to 10. Then the ten measures of y are obtained by : $y(i) = \mu_y + z(i)\,\sigma_y$ for i = 1 to 10. Finally, the ten demand points from the lognormal distribution are obtained by $d(i) = \exp(y(i))$ for i = 1 to 10. See the list below.

i	α(i)	z(i)	y(i)	d(i)
1	.05	-1.645	-1.131	0.322
2	.15	-1.036	-0.549	0.578
3	.25	-0.674	-0.202	0.816
4	.35	-0.385	0.074	1.077
5	.45	-0.125	0.322	1.380
6	.55	0.125	0.562	1.754
7	.65	0.385	0.810	2.218
8	.75	0.674	1.087	2.965
9	.85	1.036	1.433	4.192
10	.95	1.645	2.015	7.504

In a similar way, the ten IBQ points are obtained. Since the lead time is L = 2 months, the mean and standard deviation of the lead time demands are obtained as μ_{dL} = 4.92 and σ_{dL} = 4.25. The counterpart normal distributed statistics become μ_y = 1.314 and σ_y = 0.747. Now following the same steps are described above, the list below is generated.

i	α(i)	z(i)	y(i)	dL(i)= q(i)
1	.05	-1.645	0.084	1.088
2	.15	-1.036	0.539	1.715
3	.25	-0.674	0.810	2.247
4	.35	-0.385	1.026	2.790
5	.45	-0.125	1.220	3.387
6	.55	0.125	1.407	4.084
7	.65	0.385	1.601	4.958
8	.75	0.674	1.817	6.155
9	.85	1.036	2.088	8.068
10	.95	1.645	2.542	12.714

For simplicity in the tables below, the ten demand points and ten quantity points are listed in closest integers. So the integer demand points (*d*) are: 0, 1, 1, 1, 1, 2, 2, 3, 4, 8. The ten quantity points (q) become: 1, 2, 2, 3, 3, 4, 5, 6, 8, 13. Subsequently, in the computations, the fractional values are used.

The computations continue to find the optimal quantity to buy (IBQ). The parameters used in the example are: *gw* = 0.2, *sr* = 0.5, *h* = 0.24, *Co* = 24 and *mu* = 1.25. Further recall *L* = 2 months and *c* = $1356. So ,when *q* = 1.088 and *d* = 0.322, for example, the annual costs become the following: K(order) = 112.6, K(hold) = 138.0, K(goodwill) = 0.0 and K(scrap) = 0.0 and thereby K(*q,d*) = 250.6 (see Table 1 below).

The ten (average monthly) demand points (*d*) are listed in the left column (in integers), and the ten candidate initial buy quantities (*q*) are given in the first row (in integers). Recall, *d* and *q* are both shaped like a lognormal distribution. For brevity, the table lists only the integer portion of *d* and *q;* but in the computation of the cost model, the corresponding fractional values are used throughout. The body of the table gives the values of *K(q,d)* = annual cost when the initial buy quantity is *q* and the average monthly demand is *d*. The bottom row gives the values of *K(q),* the expected annual cost when the initial buy quantity is *q*. Note. the minimum of *K(q*) is 1311 and this occurs when *q* = 4. Thereby, IBQ = 4.

Table 1. K(q,d) for 10 average monthly demand points (*d*) and 10 candidate initial buy quantities (*q*) on a part with cost per unit = $1356 and lead time L = 2 months.

d/q	1	2	2	3	3	4	5	6	8	13
0	251	285	339	420	537	866	1743	2943	4862	9524
1	352	341	365	403	461	551	695	950	2291	6953
1	576	396	409	433	470	529	625	798	1174	4541
1	812	600	459	474	500	541	610	736	1012	2066
1	1078	863	687	523	541	571	622	716	924	1730
2	1398	1182	1003	825	635	615	652	723	881	1500
2	1810	1593	1412	1232	1037	815	702	753	871	1342
3	2393	2176	1994	1811	1613	1386	1108	814	898	1242
4	3368	3151	2967	2783	2582	2350	2064	1680	1086	1209
8	5916	5698	5515	5329	5125	4889	4595	4198	3574	2112
K(q)	1795	1628	1515	1423	1350	1311	1342	1431	1757	3222
						min				

Summary

A cost model is developed to determine the initial buy quantity for a new service part in a stocking location. The model uses the data and statistics from the new-parts-of-the-past and links them to the new-parts-of-the-future. The average 1-month demand for the initial year is assumed to follow the lognormal distribution. An example is presented to illustrate the model.

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