DEMANDS ALONG THE SUPPLY CHAIN

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ABSTRACT

This paper describes how the monthly demands vary at the locations along the supply chain, coming from the customers to a dealer onto a distribution center and finally to a supplier. The mean, standard deviation and coefficient of variation are measured for each of the locations. The results indicate when the demands tend to be normally distributed and when non-normal.

INTRODUCTION

Consider a dealer that carries inventory on parts to meet the oncoming customer demands. This paper assumes the monthly demands from the customers are horizontal (no trend or seasonal pattern); and also, the monthly demands at a dealer are shaped like a Poisson probability distribution. The dealer replenishes the stock on the part from a distribution center (DC), and the DC is replenished from a supplier. The replenishment quantitiy Q at a location depends mostly on the forecast of demands and the cost per unit. For convenience, the replenishment order quantity can be stated in month-in-buy (mib) terms where $Q = mib \times \mu$ and μ is the average of the monthly demand forecast at the location. When mib is 1.0, Q is one month supply; when mib is 2.0, Q is two months supply, and so forth. The aggregate flow of replenishments from all the dealers are the demands at the DCs; and the flow of replenishments from the DCs are the demands at the supplier. This paper shows how the demands along the supply chain, at the dealer, DC and the supplier are related.

The results apply for the supply chain of a part (or product) that has a single supplier, many dealers and one or more distribution centers. This arrangement is common, for example, when the items in stock are service parts that are needed for the repair and maintenance of finished-good-items; or are consumer products that are sold to meet individual customer needs.

THE BULLWHIP EFFECT

This paper links to the *Bullwhip Effect* study [2] that was introduced by J. Forrester in 1961. In the study, the demands that occur upstream in the supply chain resemble a cracking whip that affects the flow of demands downstream, and thus the term bullwhip effect is used. Demand variability increases as one moves up the supply chain away from the retail customer, and small changes in consumer demand can result in large variations in orders placed upstream. Eventually, the network can oscillate in very large swings as each organization in the supply chain seeks to solve the problem from its own perspective. An excellent description on this phenomenon is presented in [3], and a more recent discussion is found in [4]. This paper develops a way to quantify in a statistical sense the flow of demands on the main locations in the supply chain. The focus is on the monthly demands by location. For various supply chain configurations, the parameters (mean, standard deviation, coefficient-of-variation) are generated to compare the relative values from location-to-location and also to note the type of distribution (normal, non-normal) by location. The non-normal distributions usually falls into the "lumpy" category.

NOTATION

To clarify the discussion to follow, the notation used in this paper is summarized below.

d_0 = monthly demands all customers
μ_0 = the mean of d_0

N_1 = number of dealers
M_1 = month-in-buy from dealer to DC
d_1 = monthly customer demands per dealer
μ_1, σ_1 are the mean and standard deviation of d_1
c_1 = cov of d_1

d_2 = monthly demands per DC
μ_2, σ_2 are the mean and standard deviation of d_2
c_2 = cov of d_2

N_3= number of DCs
M_3 = month-in-buy from DC to supplier
d_3 = monthly demands all DCs
μ_3, σ_3 are the mean and standard deviation of d_3
c_3 = cov of d_3

d_4 = monthly demands to supplier
μ_4, σ_4 are the mean and standard deviation of d_4
c_4 = cov of d_4

MONTHLY DEMANDS ALL CUSTOMERS

In this study, we assign d_0 as the monthly demand for an item from all customers across all dealers, and further denote μ_0 as the corresponding average monthly demand of d_0. This paper gives examples where this aggregate demand are μ_0 = 10, 100 and 1000.

MONTHLY DEMANDS FOR AN AVERAGE DEALER

The demand for an average dealer is denoted as d_1, and when the number of dealers is N_1. the mean monthly demand for an average dealer (μ_1) becomes $\mu_1 = \mu_0/N_1$. Because the national demands for an item are spread over many dealers, the monthly demands for each stock-keeping-unit (SKU) at an individual dealer are typically small. For this reason, the Poisson distribution is assumed as the distribution for the monthly demands (from the customers to an individual dealer). The Poisson is also ideal for analysis since it is a one parameter distribution since the mean and variance are equal. Thereby, we assume the monthly demands coming to a dealer from its customers is Poisson with an average of μ_1 and a standard deviation of $\sigma_1 = \sqrt{\mu_1}$. Recall, the coefficient of variation (cov) of a random variable is defined as the ratio of the standard deviation over the average. So in this situation, the coefficient of variation becomes $cov_1 = \sigma_1/\mu_1$. Table 1 lists values μ_1, σ_1 and cov_1 as the average monthly demands range from 1 to 1000. Note how cov_1 is largest (1.00) when $\mu_1 = 1$ and becomes increasingly smaller as μ_1 rises. In the table, the cov extremes are 1.00 on the high end and 0.03 on the low end. Further note, when the demands are all positive and normally distributed, the cov attains a value is 0.33 or less. This is because (with all demands positive) the mean is at least three standard deviations larger than zero. On the other extreme, when the cov is one, the monthly demands are distributed as an exponential distribution, since for this distribution, the standard deviation is the same as the mean. So in essence, as the average monthly demands (from customers) increase, the monthly demands at the average dealer tends like a *normal* distribution; and as the average monthly demands go down, the monthly demands are of the lumpy type and shaped more like an *exponential* distribution.

Table 1. Monthly demand statistics for an average dealer

μ_1	σ_1	cov_1
1	1.00	1.00
5	2.24	0.45
10	3.16	0.32
50	7.07	0.14
100	10.00	0.10
500	22.36	0.04
1000	31.62	0.03

MONTHLY DEMANDS FOR AN AVERAGE DC

The monthly demands for an average distribution center (DC) is here denoted as d_2. The associated mean monthly demand is μ_2 and corresponding standard deviation is σ_2. Also, the coefficient of variation is $cov_2 = \sigma_2/\mu_2$. We compute the measures (μ_2 , σ_2, cov_2) for an average DC in the analysis given below.

ANALYSIS

When the dealer needs replenishment stock, it buys from its assigned distribution center (DC). The buy quantities (also called order quantities or purchase quantities) are in lot sizes that are economical for them. Suppose the lot size is q and this quantity is sufficient for the forecast of demands over the next mib months. In this analysis, mib = months-in-buy and represents the buy amount of replenishment stock in monthly requirements. Of interest here is to measure the mean and variance for the average monthly replenishment quantities from the dealer to the DC. For notation, E(q) = expected replenishment quantity per month and V(q) is the associated variance.

Consider the general situation when the months-in-buy, mib, is an integer of m = 1,2, 3,... Suppose the dealer demands for the most recent m months are: d(1), ..., d(m). Also assume the stock at the dealer is adequate at the first month (t = 0), and the dealer needs replenishment stock at month t = m, for a quantity size that covers the next m month requirements. So the replenishment quantity from the dealer to the DC will be q(t) = 0 for months t = 1 to m-1 and will be approximately q(m) = (d(1)+...+d(m)) at the end of month m. Since mib = m, this pattern will repeat as the months move along. Of interest now is to determine the expected value and variance of the monthly replenishment quantities, q(t). Note (m-1) of the quantities are zero, and one quantity is an m-month supply. The expected value of q is:

$$
\begin{aligned}
E(q) &= ((m-1)/m) \times 0 + 1/m \times E(d(1) + \dots + d(m)) \\
&= 1/m \times m\, E(d) \\
&= E(d).
\end{aligned}
$$

In a similar way, the expected value of q^2 becomes

$$
\begin{aligned}
E(q^2) &= 1/m\, E[(d(1)+\dots+d(m))^2] \\
&= 1/m[mE(d^2) + m(m-1)E(d)^2] \\
&= E(d^2) + (m-1)E(d)^2 \\
&= V(d)+E(d)^2 + (m-1)E(d)^2 \\
&= V(d) + mE(d)^2.
\end{aligned}
$$

Finally, the variance of q becomes

$$
\begin{aligned}
V(q) &= E(q^2) - E(q)^2 \\
&= V(d)+(m-1)E(d)^2.
\end{aligned}
$$

Returning to the notation for the average DC, the following substitutions are made:

$$
\begin{aligned}
\mu_2 &= E(q) \\
\sigma_2^2 &= V(q) \\
\mu_1 &= E(d) \\
\sigma_1^2 &= V(d).
\end{aligned}
$$

The month-in-buy from the dealer to the DC is denoted as M_1. Further, N_1 is the number of dealers and N_3 the number of DCs. The statistics for the mean monthly demands d_2 at an average DC are as below:

$$\mu_2 = (N_1/N_3)\mu_1$$

$\sigma_2^2 = (N_1/N_3)(\sigma_1^2 + [M_1 - 1]\mu_1^2)$
$cov_2 = c_2 = \sigma_2/\mu_2$.

Table 2 shows how cov_1 and M_1 = mib are related to cov_2 as cov_1 ranges from 0.1 to 1.0 and M_1 from 1 to 6 months. Recall, cov_1 is a measure of the variation in demands from the customers to an average dealer, and cov_2 is the counterpart measure of the variation in demands from the dealer to the an average DC . Note how cov_2 increases significantly as the M_1 increases beyond one month. Also note when M_1 is two or larger, cov_2 increases gradually as cov_1 goes from 0.1 to 1.0. In essence, when the mib (M1) from the dealer to the DC increases above one, the demands to the DC become lumpy.

SUMMARY STATISTICS ACROSS THE SUPPLY CHAIN

In summary, the demand statistics across the supply chain are given in Tables 3.1 and 3.2.

Table 2. Values of cov_2 as related to cov_1 and M_1

	cov_1									
M_1	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
1	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2	1.00	1.02	1.04	1.08	1.01	1.17	1.22	1.28	1.35	1.41
3	1.42	1.43	1.45	1.47	1.50	1.54	1.58	1.62	1.68	1.73
4	1.73	1.74	1.76	1.78	1.80	1.83	1.87	1.91	1.95	2.00
5	2.00	2.01	2.02	2.04	2.06	2.09	2.12	2.15	2.19	2.24
6	2.24	2.24	2.26	2.27	2.29	2.32	2.34	2.37	2.41	2.45

MONTHLY DEMANDS FOR ALL DC's

In this section, assume N_3 is the number of DCs and d_3 is the average demand for all DCs. Further μ_3 is the average monthly demand, σ_3 the standard deviation and c_3 is the coefficient of variation of d_3. So now, $\mu_3 = N_3 \times \mu_2$, $\sigma_3^2 = N_3 \times \sigma_2^2$ and $cov_3 = c_3 = \sigma_3/\mu_3$.

MONTHLY DEMANDS FOR THE SUPPLIER

The monthly demands for the supplier is here denoted as d_4. In the same way, μ_4 is the associated average monthly demand, σ_4 the standard deviation and c_4 is the coefficient of variation of d_4. In the calculations, M_3 is the month-in-buy from the DC to the supplier.

So now,
$\mu_4 = \mu_3$
$\sigma_4^2 = \sigma_3^2 + (M_3 - 1)\mu_3^2$
$c_4 = \sigma_4/\mu_4$.

Table 3.1, Average monthly demands (μ) and cov (c) for all customers, per dealer, per DC, all DC's and supplier when number of dealers is $N_1 = 100$ and number of DC's is $N_3 = 1$.

M_1	M_3	All μ_0	per dealer μ_1	per dealer c_1	per DC μ_2	per DC c_2	all DCs μ_3	all DCs c_3	supplier μ_4	supplier c_4
1	1	10	0.1	3.16	10.0	0.32	10.0	0.32	10.0	0.32
1	1	100	1.0	1.00	100.0	0.10	100.0	0.10	100.0	0.10
1	1	1000	10.0	0.32	1000.0	0.03	1000.0	0.03	1000.0	0.03
1	2	10	0.1	3.16	10.0	0.32	10.0	0.32	10.0	1.05
1	2	100	1.0	1.00	100.0	0.10	100.0	0.10	100.0	1.00
1	2	1000	10.0	0.32	1000.0	0.03	1000.0	0.03	1000.0	1.00
2	1	10	0.1	3.16	10.0	0.33	10.0	0.33	10.0	0.33
2	1	100	1.0	1.00	100.0	0.14	100.0	0.14	100.0	0.14
2	1	1000	10.0	0.32	1000.0	0.10	1000.0	0.10	1000.0	0.10
2	2	10	0.1	3.16	10.0	0.33	10.0	0.33	10.0	1.05
2	2	100	1.0	1.00	100.0	0.14	100.0	0.14	100.0	1.01
2	2	1000	10.0	0.32	1000.0	0.10	1000.0	0.10	1000.0	1.01

Table 3.2, Average monthly demands (μ) and cov (c) for all customers, per dealer, per DC, all DC's and supplier when number of dealers is $N_1 = 1000$ and number of DC's is $N_3 = 5$.

M_1	M_3	All μ_0	per dealer μ_1	per dealer c_1	per DC μ_2	per DC c_2	all DCs μ_3	all DCs c_3	supplier μ_4	supplier c_4
1	1	10	0.0	10.00	2.0	0.71	10.0	0.32	10.0	0.32
1	1	100	0.1	3.16	20.0	0.22	100.0	0.10	100.0	0.10
1	1	1000	1.0	1.00	200.0	0.07	1000.0	0.03	1000.0	0.03
1	2	10	0.0	10.00	2.0	0.71	10.0	0.32	10.0	1.05
1	2	100	0.1	3.16	20.0	0.22	100.0	0.10	100.0	1.00
1	2	1000	1.0	1.00	200.0	0.07	1000.0	0.03	1000.0	1.00
2	1	10	0.0	10.00	2.0	0.71	10.0	0.32	10.0	0.32
2	1	100	0.1	3.16	20.0	0.23	100.0	0.10	100.0	0.10
2	1	1000	1.0	1.00	200.0	0.10	1000.0	0.04	1000.0	0.04
2	2	10	0.0	10.00	2.0	0.71	10.0	0.32	10.0	1.05
2	2	100	0.1	3.16	20.0	0.23	100.0	0.10	100.0	1.01
2	2	1000	1.0	1.00	200.0	0.10	1000.0	0.04	1000.0	1.00

CONCLUSIONS

For a part in a stocking location (dealer, DC), the replenishment measures (order point and order level) are typically computed using the desired service level, lead time and order size, with the assumption that the monthly demands follow the normal distribution as described in references [1], [5], [6].

A difficulty occurs when the normal distribution does not apply. Reference [5] shows how to compute the safety stock (with order point and order level) when the monthly demands are not normally distributed, but instead are of the lumpy type. The truncated normal distribution is used for this purpose. In general, when the safety stock is computed by incorrectly assuming a normal distribution instead of the true lumpy distribution, the safety stock is not large enough. Hence, the actual service level for a lumpy item will not achieve the desired service level that is sought in the computations. Tables 3.1 and 3.2 show when the monthly demands tend to be normal and when they tend to be lumpy. When cov is less than 0.33, the normal applies, and when cov is one or larger, the lumpy demands apply.

Table 3.1 depicts a supply chain when100 dealers and one DC. This scenario is somewhat like a large grocery chain in a large metropolitan area. Note how the monthly demands at the dealer are often of the lumpy type. The table also shows how the monthly demands at the DC tend to be normally shaped, and the demands at the supplier are a mixture of normal and lumpy type, depending on the month-in-buy from the DC.

Table 3.2 gives measures of a supply chain when1000 dealers and 5 DC's. This scenario is like a service parts distribution network for an OEM that covers the total country. The table shows that the monthly demands at a dealer are not normally distributed but are of the lumpy type. Further, the monthly demands going to the DCs tend to be normally shaped, and finally, the demands going to the supplier (from the DC) are of the lumpy type when the dealer replenishment quantities are for two or more months of supply.

SUMMARY

This paper shows how the demand flows from the customers to the dealers to the DC's and to the supplier. The demand measures are the average monthly demand, the standard deviation and the coefficient of variation. The tables shows how the shape of the monthly demands can range from normal to exponential and beyond. Recall, when cov ≤ 0.33, the monthly demands could be shaped like a normal distribution, and when cov ≥ 1.00, the monthly demands are the lumpy type.

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