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Some statistical measures on the national, distribution center and dealer demands along the supply chain

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Abstract

The monthly demands in three levels of the supply chain are measured using the coefficient of variation. The supply chain here includes the national, the distribution centers and the dealers. One table compares the national demands with distribution center demands, and another compares the national demands with the dealer demands.

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Introduction

Consider a supply chain for an OEM where stock is ordered from suppliers and is stored in one or more distribution centers. The distribution centers are the source of the stock for a network of dealers; and the dealers are the source of stock to the end customers. The purpose of this paper is to study some of the statistical properties of the monthly demands at the distribution centers and at the dealers. These are compared to the monthly demands on an aggregate basis from all distribution centers, which are here called the national monthly demands.

The statistical properties of interest in this paper are the mean and variance of the monthly demands. A relative metric to observe the properties is by way of the coefficient of variation (cov) which gives the ratio of the size of standard deviation (square root of variance) over the size of the mean.

The supply chain

In this paper, the supply chain for the OEM begins where the suppliers ship inventory to the distribution center(s) who stock-to-order for the dealers and the dealers sell the stock to their customers. In service parts, the dealers also hold stock for their service repair facilities where they perform maintenance and repair on their customer's finished goods, (cars, trucks, appliances).

The demands at the distribution centers.

In the typical situation, an OEM carries stock in one or more distribution centers. Each distribution center has an assigned territory and the dealers within the territory are their customers. So the demands in a distribution center are all the orders coming from the dealers in the territory.

The demands over all distribution centers

For convenience, the sum of the demands over all of the distribution centers is here called the national demand. This is the demand that is viewed when placing orders with the suppliers to replenish the stock at the distribution centers.

The demands at the dealers

The dealers carry stock to fill the demands from their customers. In service parts, the customer demands include over-the-counter sales as well as the maintenance and repair of finished goods. In the automotive industry, the dealer also can act as the supplier to large fleets that are located nearby the dealer.

The statistical properties on the demands

As stated above, the purpose of this paper is to show some of the statistical properties on the demands at the various levels of activity in the supply chain. This paper assumes the monthly demands at the distribution centers and dealers follow a binomial distribution.

Recall in the binomial distribution, the random variable is a discrete positive integer and the parameters are the number of trials and the probability of a success per trial. The statistical properties of the binomial distribution are well known and can be found in almost any book on Statistics or Probability; see for example[1]. However, an exception to the binomial distribution is needed in the demand context of this analysis. The exception concerns the parameter that calls for the number of trials. In this situation the number of trials is not known for certain; and in itself is a random variable with a known mean and variance. The statistical properties for this latter distribution has been presented by Thomopoulos[2] where he shows how to determine the safety stock for individual stockkeeping units (sku) when a part has a multiple number of sku's. For convenience, a quick review on the statistical properties for both of the above situations on the binomial distribution are given below.

When x is binomial with parameters p and n.

Here n is the number of trials and p is the probability of a success per trial. The random variable x represents the number of successes in n trials and is a discrete integer and can range from 0 to n. The expected value (mean) and variance of x are:

$$E(x) = np$$
$$V(x) = np(1-p).$$

When x is binomial with parameter p and n is a random variable.

As above p is the probability of a success per trial. The difference from the binomial distribution, however, is where the number of trials n is a random variable. The expected value and variance of n are denoted as follows:

$$E(n) = \mu$$
$$V(n) = \sigma^2.$$

The mean and variance of x, labeled as E(x) and V(x), are obtained as below. Using conditional probability notation:

$$E(x|n) = np$$
$$E(x) = E(n)p = \mu p\ .$$

$$V(x|n) = E(x^2|n) – E(x|n)^2 = np(1-p)$$
$$E(x^2|n) = V(x|n) + E(x|n)^2 = p(1-p)n + p^2n^2$$
$$E(x^2) = p(1-p)E(n) + p^2E(n^2) = p(1-p)\mu + p^2(\mu^2 + \sigma^2)$$
$$V(x) = E(x^2) – E(x)^2 = p(1-p)\mu + p^2 \sigma^2$$

In summary,

$$E(x) = p\mu$$
$$V(x) = p(1-p)\mu + p^2 \sigma^2$$

Using the above results, it is now possible to study the corresponding statistical properties for the three type of demands defined earlier: national, distribution center and dealer demands. With each of the three type of demands, the coefficient of variation is of interest. This is a relative measure on the size of the standard deviation with respect to the size of the mean.

National demands

In this paper D_N represents the monthly national demands for a part number. This is the sum of all distribution center demands. The mean and variance of this demand is denoted by:

$$E(D_N) = \mu_N$$
$$V(D_N) = \sigma_N^2.$$

The associated coefficient of variation is

$$cov_N = \sigma_N/\mu_N$$

Distribution center demands

Let D_{DC} represent the monthly demands at a distribution center. Also let n_{DC} denote the number of distribution centers in the national system. So the average portion of the national demand that falls in a distribution center becomes

$$p_{DC} = 1/ n_{DC} .$$

Consider the binomial distribution results above with the number of trials as a random variable. Here D_{DC} is the random variable of interest and D_{DC} replaces x in the binomial notation above. Also $p_{DC} = p$, $\mu_N = \mu$, and $\sigma_N = \sigma$. So now, the mean and variance of D_{DC} for the average distribution center becomes:

$$E(D_{DC}) = \mu_{DC} = p_{DC}\,\mu_N$$
$$V(D_{DC}) = \sigma_{DC}^2 = p_{DC}\,(1 - p_{DC})\mu_N + (p_{DC}\sigma_N)^2.$$

The associated coefficient of variation is

$$cov_{DC} = \sigma_{DC}/\mu_{DC}.$$

Dealer demands

For the dealers, we let D_d represents the monthly dealer demands for the part number, and n_d the number of dealers that are active on the national level. The average portion of the national demand that is associated with a dealer becomes

$$p_d = 1/ n_d .$$

As above (with the distribution centers), we use the binomial distribution results when the number of trials is a random variable. For the dealers, D_d replaces x. Also $p_d = p$, μ_N

$= \mu$ and $\sigma_N = \sigma$. The mean, variance and coefficient of variation of D_d for an average dealer are as below:

$E(D_d) = \mu_d = p_d \mu_N$ and
$V(D_d) = \sigma_d^2 = p_d (1 - p_d)\mu_N + (p_d\sigma_N)^2$
$cov_d = \sigma_d / \mu_d$

Some comparisons.

Table 1 lists some comparisons between national demands and distribution centers demands. The national monthly demands have a cov ranging from 0.10 to 0.30, and have average monthly demands spanning 10 to 10,000. The number of distribution centers are from 2 to 15. Note, the larger the national monthly demands, the smaller the increase in the cov at the distribution centers.

Table 2 lists some comparisons between national demands and dealer demands. Again the national monthly demands have cov's from 0.10 to 0.30 and average monthly demands from 10 to 10,000. The number of dealers in the network are from 25 to 1,000. Note, how the cov_{DC} at the dealers increases relative to cov_N as the number of dealers in the network increases and as the average monthly national demand decreases.

Summary

The paper shows how to measure the cov for the monthly demands at the distribution centers and at the dealers. Table values are given to compare the cov's with corresponding measures from the monthly demands on a national level. The results of this paper are directly useful to the inventory management in planning the safety stock needs along the downstream locations of the supply chain; see e.g. [3,4,5]. The results show where the cov at the dealer level is higher than the corresponding cov at the distribution center level and this is higher than at the national level.

Table 1. Comparing the cov between national demands and distribution center demands

National cov_N	μ_N	number of distribution centers (n_{DC}) 2	3	4	5	10	15
		cov_{DC}					
0.10	10	0.33	0.46	0.56	0.64	0.95	1.19
	100	0.14	0.17	0.20	0.22	0.32	0.39
	1000	0.10	0.11	0.11	0.12	0.14	0.15
	10000	0.10	0.10	0.10	0.10	0.10	0.11
0.20	10	0.37	0.49	0.58	0.66	0.97	1.20
	100	0.22	0.24	0.26	0.28	0.36	0.42
	1000	0.20	0.20	0.21	0.21	0.22	0.23
	10000	0.20	0.20	0.20	0.20	0.20	0.20
0.30	10	0.44	0.54	0.62	0.70	0.99	1.22
	100	0.32	0.33	0.35	0.36	0.42	0.48
	1000	0.30	0.30	0.30	0.31	0.31	0.32
	10000	0.30	0.30	0.30	0.30	0.30	0.30

Table 2. Comparing the cov between national demands and dealer demands

National cov_N	μ_N	number of dealers (n_d) 25	50	100	250	500	1000
		cov_d					
0.10	10	1.55	2.22	3.15	4.99	7.06	10.00
	100	0.50	0.71	1.00	1.58	2.24	3.16
	1000	0.18	0.24	0.33	0.51	0.71	1.00
	10000	0.11	0.12	0.14	0.19	0.24	0.33
0.20	10	1.56	2.22	3.15	4.99	7.07	10.00
	100	0.53	0.73	1.01	1.59	2.24	3.17
	1000	0.25	0.30	0.37	0.54	0.73	1.02
	10000	0.21	0.21	0.22	0.25	0.30	0.37
0.30	10	1.58	2.23	3.16	5.00	7.07	10.00
	100	0.57	0.76	1.04	1.61	2.25	3.17
	1000	0.34	0.37	0.43	0.58	0.77	1.04
	10000	0.30	0.31	0.32	0.34	0.37	0.44

References:

[1]David R. Anderson, Dennis J. Sweeney and Thomas A. Williams, *Statistics for Business and Economics*, (St. Paul, Mn: West Publishing Company., 1990)

[2]Nick T. Thomopoulos, *Strategic Inventory Management and Planning.* (Carol Stream, Il: Hitchcock Publishing Co., !990)

[4]Robert G. Brown, *Smoothing, Forecasting and Prediction of Discrete Time Series*. (Englewood Cliffs, N.J: Prentice Hall, Inc., 1963)

[4]Same reference as Note 2.

[5]Nick T. Thomopoulos,. *Applied Forecasting Methods*. (Englewood Cliffs, N.J: Prentice Hall, Inc., 1980)