Supplier Lateness, Service Level and Safety Time

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Abstract *In a typical inventory holding location along the supply chain, safety stock is needed to yield a desired service level of the type (demand filled)/(total demand). To determine the safety stock, the computations assume a planned lead time provided by the supplier. In reality, however, the actual delivery time may vary from the planned lead time and often is longer. The paper explores how the achieved service level is effected when the delivery time varies in this way. The paper also shows how to determine the safety time stock needed to offset the longer than expected lead times.*

This paper concerns those locations along the supply chain that stock inventory to meet oncoming demands of the uncertain type. Customer orders arrive without advance notice and require immediate fill. The uncertainty can be measured by way of the forecast error. Another level of uncertainty is with the lead time needed to procure stock from the supplier. The actual lead time may differ from the planned lead time. The management generally sets a desired service level and seeks to have the proper amount of stock to reach this desired level. This requires finding the amount of safety stock to hold to offset the forecast error that can occur over the planned supplier lead time duration. But when the lead time is larger than the planned lead time, the service level could suffer. This paper gives a way to measure the change in the service level and also shows how an extra layer of stock (safety time stock) can be provided to offset the lead time uncertainty.

Two key measures on the performance of the inventory system are the amount of safety stock provided and the service level attained. The service level (SL) is typically measured as the ratio of (demand filled) over (total demand). The safety stock is the stock carried to meet the uncertainty associated with the forecast of the demands over the lead time duration The uncertainty in demands is measured by the standard deviation of the one month ahead forecast error and is denoted here as σ. The safety stock also depends on the lead time provided by the supplier.

Safety stock for a desired service level

References [1] [2] [3] show how to compute the safety stock to yield the service level goal as desired by the management. On an individual part, the data used to determine the safety stock is listed below:

SL = desired service level
F = average monthly forecast
L = lead time to procure the part from the supplier
Q = the size of the order quantity
σ = the standard deviation of the one-month ahead forecast error

Note the lead time forecast is measured by $F_L = LxF$ and the corresponding lead time standard deviation is $\sigma_L = \sqrt{L}\,\sigma$. To determine how much safety stock is needed, the partial expectation is computed from $E(k) = (1\text{-}SL)Q/\sigma_L$. Now E(k) yields the safety factor k by way of a table lookup as described in reference [3]. The safety stock becomes $SS = k\,\sigma_L$. These measures are used to find the order point and order level. The order point is $OP = F_L + SS$ and the order level is OL = OP + Q. The buying rule is when the on-hand plus on-order (OH+OO) inventory is at OP or lower, then the buy quantity = OL - (OH+OO).

For the analysis of this paper, references [2] [3] show how the order size and forecast error are converted in units of the average monthly forecast as shown below:

M = Q/F = months-in-buy
cov = σ/F = coefficient of variation

This way, the data is independent from the forecast size and is defined in relative terms. The data to determine the safety stock is now reduced to the following:

SL = desired service level
L = lead time in months
M = the order size in months supply
cov = the coefficient of variation

The procedure to find the safety stock in months supply is as follows. First, the partial expectation becomes E(k) = (1-SL)M/($\sqrt{L}$ cov). Second, a table lookup yields the safety factor k. So now, thirdly, the safety stock in months is computed by k$\sqrt{L}$ cov. Note, the lead time forecast is L months and the order point (in months supply) is OP = L + k$\sqrt{L}$ cov.

Service level when the actual delivery time is w instead of L

Suppose a situation with desired service level SL, and where the safety stock, lead time forecast and the order point from the planned lead time L are computed as shown above. But assume the actual (delivery) lead time is w instead of L. How does this affect the actual service level? Note, the delivery time forecast becomes F_w = wxF and the delivery time standard deviation is $\sigma_w = \sqrt{w}\,\sigma$. So the associated safety factor is measured from k = (OP – F_w)/σ_w Using a table lookup, k gives the corresponding partial expectation E(k), whereby the conditional service level (SL|w) = 1 – E(k)σ_w/Q.

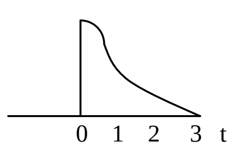
Expected value of the service level

Reference [3] pages 243-259, shows how the conditional service level (SL|w) is used to find the expected service level when the delivery time follows a normal distribution with mean L. This paper expands the analysis to another related distribution. In general, when the probability distribution of w is f(w), the expected service level is obtained from the integral E(SL) = $\int$(SL|w) f(w)dw .

The Half-Normal distribution

Recall the standard normal distribution where the variable z is N(0,1) with $\mu_z = 0$ and $\sigma_z = 1$. Reference [2] explores the left-truncated normal where the only values of z that occur are those when z>k. The variable t = z - k is used and t >0. The left-truncated distribution becomes the half-normal when k = 0. When k=0, the reference shows where the expected value is E(t) = 0.7979 and the standard deviation is σ(t) = 0.6078. For the half-normal, E(t) ~ 0.8 σ_z. Note the standard deviation of a full normal and the mean of a half normal are related as: σ_z = 1.25E(t). Figure 1 depicts the half-normal distribution.

Figure 1. The half-normal distribution



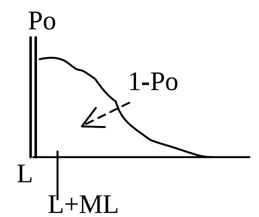
The delivery time distribution

This paper assumes the delivery time (denoted as w) is a mixed discrete and continuous variable where Po = P(w = L), (1- Po) = P(w > L) and w begins at the mode where w = L. The variable w is shaped like the right-hand side of the normal distribution. For convenience, w is assumed in monthly units and ML = expected months late. The values of w begin at the mode L and spans to the right and seldom falls beyond L + 3x1.25ML. The distribution is the following and is depicted in Figure 2.

f(w) = Po at w = L

Half-normal at $w > L$

Figure 2. The lead time distribution



Lead time sensitivity on the service level

To illustrate how the lead time affects the service level, consider the following scenario when the planned lead time is L = 3.00 months, but the delivery times vary from 3.00 to 3.75 months and thereby the late time ranges fro 0.00 to 0.75 months. The computations assume the average monthly forecast is F = 100 and the associated standard deviation is $\sigma = 30$, hence, cov = 0.30. Further, the order quantity is Q = 100 and thereby M = 1.00. The table lists the conditional service levels SL|w for selected values of w.

w	3.00	3.15	3.30	3.45	3.60	3.75
late	0.00	0.15	0.30	0.45	0.60	0.75
SL|w	0.950	0.915	0.861	0.792	0.709	0.640

Note how the achieved service level drops as the delivery time increases above the planned lead time L.

The safety time and safety time stock

Safety time (ST) is used as a parameter that allows the inventory management to hold added stock to offset the increase in delivery time. When the safety time is one or less (months), the safety time stock (STS) for month t is computed by $STS_t = ST \text{x} F_{t+1}$ where F_{t+1} is the forecast for month (t+1), and so forth. The total safety stock for a part in month t becomes $TSS_t = SS + STS_t$. In the examples of this paper, the forecasts are assumed horizontal and so STS = STxF is used throughout. In this way, the order point becomes $OP = TSS + F_L$.

When the safety time is ST and the months late is ML, the average delivery time becomes w = (L + ML). Further, the delivery-time forecast is $F_w = w \text{x} F$ and the associated standard deviation is σ_w. So now, the safety factor becomes $k = (OP + STS - F_w)/\sigma_w$ and the corresponding partial expectation,E(k), gives SL(Po,ST|w) = 1 – $E(k)Q/\sigma_w$.

With Po and ST, the service level is obtained as follows:

$$SL(Po,ST) = SL(Po,ST|w = L)Po + SL(Po,ST|w = L + ML)(1 - Po)$$

The six scenarios listed below are used to measure the sensitivity of the service level with the supplier lateness and the safety time. The results are listed in Tables 1 to 6. With each table, the rows pertain to the portion of deliveries not late, Po (0.0 to 1.0). The columns identify the safety time, ST = kx1.25ML (0.0 to 2.0) . The cells in the body give the service level achieved with each combination of Po and ST. The right-hand column lists the safety factor, k', that yields the minimum safety time needed to bring the achieved service level, SL(Po,ST), up to the same as the desired service level, (SL).

scenarios	SL	cov	M	L	ML
1	.90	0.3	0.25	0.25	0.10
2	.90	0.5	0.50	0.50	0.20
3	.95	0.3	1.00	1.00	0.40
4	.95	0.5	1.00	3.00	0.50
5	.97	0.3	1.00	1.00	0.40
6	.97	0.5	1.00	3.00	0.50

Table 1 SL(Po,ST) when SL = .90, cov = 0.30
M = 0.25, L = 0.25, ML = 0.10, ST = k(1.25ML)

Po \ k	0.0	0.5	1.0	1.5	2.0	k'
0.00	0.699	0.811	0.891	0.942	0.972	1.09
0.10	0.719	0.825	0.899	0.947	0.974	1.01
0.20	0.739	0.839	0.908	0.952	0.977	0.94
0.30	0.759	0.854	0.917	0.957	0.979	0.86
0.40	0.779	0.868	0.926	0.962	0.982	0.78
0.50	0.799	0.882	0.935	0.967	0.985	0.67
0.60	0.820	0.896	0.944	0.972	0.987	0.54
0.70	0.840	0.910	0.953	0.977	0.990	0.43
0.80	0.860	0.924	0.962	0.982	0.992	0.31
0.90	0.880	0.938	0.971	0.988	0.995	0.17
1.00	0.900	0.952	0.980	0.993	0.998	0.00

Table 2 SL(Po,ST) when SL = .90, cov = 0.50
M = 0.50, L = 0.50, ML = 0.20, ST = k(1.25ML)

Po \ k	0.0	0.5	1.0	1.5	2.0	k'
0.00	0.714	0.812	0.884	0.932	0.963	1.17
0.10	0.732	0.826	0.893	0.938	0.966	1.08
0.20	0.751	0.839	0.902	0.944	0.970	0.98
0.30	0.770	0.853	0.911	0.949	0.973	0.90
0.40	0.788	0.866	0.920	0.955	0.976	0.81
0.50	0.807	0.880	0.929	0.961	0.979	0.70
0.60	0.826	0.893	0.938	0.966	0.983	0.57
0.70	0.844	0.907	0.948	0.972	0.986	0.44
0.80	0.863	0.920	0.957	0.978	0.989	0.32
0.90	0.881	0.934	0.966	0.983	0.992	0.18
1.00	0.900	0.948	0.975	0.989	0.996	0.00

Table 3 SL(Po,ST) when SL = .95, cov = 0.30
M = 1.00, L = 1.00, ML = 0.40, ST = k(1.25ML)

Po \ k	0.0	0.5	1.0	1.5	2.0	k'
0.00	0.723	0.874	0.957	0.990	0.998	0.96
0.10	0.746	0.886	0.961	0.991	0.998	0.93
0.20	0.769	0.897	0.965	0.992	0.999	0.89
0.30	0.791	0.909	0.970	0.993	0.999	0.84
0.40	0.814	0.920	0.974	0.994	0.999	0.78
0.50	0.837	0.932	0.978	0.995	0.999	0.70
0.60	0.859	0.944	0.982	0.996	0.999	0.58
0.70	0.882	0.955	0.986	0.997	0.999	0.46
0.80	0.905	0.967	0.990	0.998	1.000	0.37
0.90	0.927	0.978	0.995	0.999	1.000	0.22
1.00	0.950	0.990	0.999	1.000	1.000	0.00

Table 4 SL(Po,ST) when SL = .95, cov = 0.50
M = 1.00, L = 3.00, ML = 0.50, ST = k(1.25ML)

Po \ k	0.0	0.5	1.0	1.5	2.0	k'
0.00	0.833	0.906	0.951	0.976	0.990	0.99
0.10	0.845	0.913	0.955	0.978	0.991	0.94
0.20	0.856	0.920	0.959	0.980	0.992	0.89
0.30	0.868	0.927	0.963	0.983	0.992	0.82
0.40	0.880	0.934	0.967	0.985	0.993	0.74
0.50	0.891	0.942	0.971	0.987	0.994	0.64
0.60	0.903	0.949	0.975	0.989	0.995	0.53
0.70	0.915	0.956	0.979	0.991	0.996	0.43
0.80	0.927	0.963	0.983	0.993	0.997	0.32
0.90	0.938	0.970	0.987	0.995	0.998	0.18
1.00	0.950	0.977	0.991	0.997	0.999	0.00

Table 5 SL(Po,ST) when SL = .97, cov = 0.30
M = 1.00, L = 1.00, ML = 0.40, ST = k(1.25ML)

Po \ k	0.0	0.5	1.0	1.5	2.0	k'
0.00	0.784	0.911	0.973	0.994	0.999	0.98
0.10	0.803	0.919	0.976	0.995	0.999	0.95
0.20	0.821	0.928	0.978	0.995	0.999	0.92
0.30	0.840	0.936	0.981	0.996	0.999	0.88
0.40	0.859	0.944	0.984	0.997	1.000	0.83
0.50	0.877	0.953	0.986	0.997	1.000	0.76
0.60	0.896	0.961	0.989	0.998	1.000	0.66
0.70	0.914	0.970	0.992	0.998	1.000	0.51
0.80	0.933	0.978	0.994	0.999	1.000	0.41
0.90	0.951	0.987	0.997	0.999	1.000	0.26
1.00	0.970	0.995	1.000	1.000	1.000	0.00

Table 6 SL(Po,ST) when SL = .97, cov = 0.50
M = 1.00, L = 3.00, ML = 0.50, ST = k(1.25ML)

Po \ k	0.0	0.5	1.0	1.5	2.0	k'
0.00	0.885	0.938	0.970	0.986	0.994	1.01
0.10	0.894	0.943	0.972	0.988	0.995	0.96
0.20	0.902	0.948	0.975	0.989	0.995	0.91
0.30	0.911	0.953	0.977	0.990	0.996	0.85
0.40	0.919	0.958	0.980	0.991	0.996	0.77
0.50	0.928	0.963	0.982	0.992	0.997	0.68
0.60	0.936	0.968	0.985	0.994	0.997	0.57
0.70	0.945	0.973	0.987	0.995	0.998	0.45
0.80	0.953	0.978	0.990	0.996	0.998	0.35
0.90	0.962	0.982	0.993	0.997	0.999	0.20
1.00	0.970	0.987	0.995	0.998	0.999	0.00

Table 7 summarizes the minimum safety factors obtained from the right-hand columns of Tables 1 to 6. Recall these factors give the minimum safety time needed to bring the achieved service level up to the desired service level. In general, the safety factors needed for each Po are relatively consistent. The right-hand column gives the average safety factor for each Po. The safety factors are used to generate the quadratic fit listed below. The regression fit yelds a correlation higher than 0.99.

$$k = 1.022 - 0.360Po - 0.642Po^2$$

The results can be applied to the parts in the inventory by the following steps:

1. For each vendor that supplies parts to the inventory, measure Po and ML from the past deliveries.
2. Use Po and the quadratic equation to find the minimum safety factor, k, for the vendor.
3. Use ML and k to determine the safety time, ST, for the vendor.
4. For each part of the vendor, apply the safety time with the forecasts to yield the safety time stock, STS.
5. For each part, apply the regular safety stock, SS, with the safety time stock, STS, to get the total safety stock, TSS = SS + STS.
6. The order point for the part becomes OP = F_L + TSS.

Table 7. Minimum safety factors from Scenarios 1 to 6

Po	1	2	3	4	5	6	average
0.00	1.09	1.17	0.96	0.99	0.98	1.01	1.03
0.10	1.01	1.08	0.93	0.94	0.95	0.96	.98
0.20	0.94	0.98	0.89	0.89	0.92	0.91	.92
0.30	0.86	0.90	0.84	0.82	0.88	0.85	.84
0.40	0.78	0.81	0.78	0.74	0.83	0.77	.78
0.50	0.67	0.70	0.70	0.64	0.76	0.68	.69
0.60	0.54	0.57	0.58	0.53	0.66	0.57	.57
0.70	0.43	0.44	0.46	0.43	0.51	0.45	.45
0.80	0.31	0.32	0.37	0.32	0.41	0.35	.33
0.90	0.17	0.18	0.22	0.18	0.26	0.20	.20
1.00	0.00	0.00	0.00	0.00	0.00	0.00	.00

Conclusions

This paper shows how the attained service level may differ from the desired service level when the actual (delivery) lead time is different from the planned lead time. Measures on the achieved service level are listed for different values of the desired service level, the coefficient of variation on the forecast error, the planned lead time, the buy quantity and the measure of late lead times. The paper introduces a parameter called the safety time and shows how it may be used to provide added stock to offset the uncertainty in the delivery time. This way the desired service level may be achieved as planned in spite of variations in the lead time.

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